Gravitational waves interacting with a spinning charged particle in the presence of a uniform magnetic field

D. B. Papadopoulos*
Department of Physics,
Section of Astrophysics, Astronomy and Mechanics,
Aristotelian University of Thessaloniki,
54124 Thessaloniki, GREECE

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Abstract

The equations which determine the response of a spinning charged particle moving in a uniform magnetic field to an incident gravitational wave are derived in the linearized approximation to general relativity. We verify that 1) the components of the 4-momentum, 4-velocity and the components of the spinning tensor, both electric and magnetic moments, exhibit resonances and 2) the co-existence of the uniform magnetic field and the GW are responsible for the resonances appearing in our equations. In the absence of the GW, the magnetic field and the components of the spin tensor decouple and the magnetic resonances disappear.

1 Introduction

In recent years, there exists an increasing interest in problems related to the motion of spinning particles in the presence of external fields (e.g. gravitational and electromagnetic fields). Thus, some interesting results related to this topic have been discussed by Mohseni et. al.[1,2,3]. Actually, they have discussed the motion of a classical spinning particle in the field of a weak gravitational wave (GW) and they found that the characteristic dimensions of the particle's orbits are sensitive to the ratio of the spin to the mass of the particle.

The problem of spinning particle(s), in the frame of special relativity, has been discussed by Frenkel[4] and later on, Bargmann et.al.[5] extended Frenkel's work for

*E-mail address:papadop@astro.auth.gr

a spinning particle in the presence of an electromagnetic field. Karl Yee et al.[6] have obtained the equations of motion for the position and spin of a classical particle coupled to an external electromagnetic and gravitational potential from an action principle.

The discussion of a spinning particle in the frame of general relativity goes back to Mathisson [7] and Papapetrou [8] (For a recent discussion on Mathisson's spinning electron equations see also Horvathy [9]). Neglecting self-gravitation and back reaction they have endowed a particle with spin by considering a rotating mass energy distribution in the limit of vanishing volume but with the angular momentum remaining finite. Later on the theory was clarified by Dixon [10] and developed by Ehlers and Rudolph [11].

Cho[12], using an energy-momentum tensor for spinning particles due to Dixon and Bailey-Israel[13], developed the post-Newtonian approximation for n spinning particles in a self-contained manner following closely the procedure presented in the well known text book by Weinberg [14].

The motion of a classical charged massive spinning particle in the frame of general relativity in the presence of an external electromagnetic and gravitational field was extensively investigated by Dixon [15,16] and Souriau [17,18,19]. The dynamics of spinning test particles which are close to massive compact objects was investigated by several authors; this kind of investigation includes, generation of gravitational waves as spinning particles fall into black holes [20,21,22], chaotic behavior of spinning particles in certain space times [23,24,25] and numerical computations for orbital motions in Kerr background [26]. Interesting results regarding the interaction of spinning test matter with gravitational and electromagnetic waves have been obtained by studying the classical motion of the spinning test particles in plane gravitational and electromagnetic field solution to the Einstein-Maxwell equations [27]. Plane-fronted gravitational waves (pp-waves) have received some attention recently because of their high symmetry and the fact that gravitational plane waves are a subclass of pp-waves [28,2]. They are assumed to describe the gravitational field at great distances from the radiating source and they can be purely gravitational, purely electromagnetic or both, depending on the source. In such backgrounds scattering effects of spinning particles have been discussed from a different point of view by several authors [27,28,29].

Using Grassmann variables Barducci et. al.[30], Ravndal [31] and Rietdijk and van Holten[32] have obtained several interesting results related to the spinning particles. Actually, in a paper by van Holten [33], the constrain $p^a S_{ab} = 0$, is satisfied by expressing the spin tensor S_{ab} as a product of two Grassmann variables. In this case, the equations of motions are derived using a Dirac-Poisson brackets formalism.

In this paper, we consider a charged massive spinning particle, in the Dixon-Souriau(DS) model[21], which includes spin-gravity terms and spin-electromagnetism terms as well (for an uncharged particle the model reduces to the well known Papapetrou one). Following the strategy of Ref.(1), in the so called DS equations of motion and neglecting the spin-electromagnetism interaction terms [21,35], we determine the

response of a spinning charged particle in a uniform magnetic field to an incident GW and verify a strong coupling of the external fields (magnetic and gravitational fields) to the spin, producing resonaces. Those resonances may cause drastic enhancement in motion which may have interesting astrophysical applications.

The paper is organized as follows: In Section 2, we present the so called DS equation of motion and describe the components of the GW which disturbs the Minkowski space time with signature $(-1, 1, 1, 1)$. In Section 3, we consider only the case where $\lambda = 0$. In other words, in the DS equations we neglect the spin-electromagnetic interaction terms and in the frame of linearized theory of gravity we obtain, for the spinning charged particle, solutions for the components of the 4-momentum, its 4-velocity and its space components x^{μ} in a coordinate system (t, x, y, z). In Section 4, we discuss the obtained results.

2 The DS Equations of Motion

The equations of motion of a spinning test particle originally derived from Papapetrou [8] and later on reformulated by Dixon [10,15,16].

Souriau [18,19] derived the so called DS equations of motion of a spinning test particle with charge e in a given gravitational and electromagnetic background. These equations are [21]:

$$\frac{dx^{\mu}}{d\tau} = \upsilon^{\mu} \tag{1}$$

$$\frac{dp^{\mu}}{d\tau} = -\Gamma^{\mu}_{\lambda\nu}\upsilon^{\lambda}p^{\nu} - \frac{1}{2}R^{\mu}_{\nu\lambda\rho}S^{\lambda\rho}\upsilon^{\nu} + eF^{\mu}_{\beta}\upsilon^{\beta} - \frac{\lambda}{2}S^{\kappa\rho}\nabla^{\mu}F_{\kappa\rho} \tag{2}$$

$$\frac{dS^{\mu\nu}}{d\tau} = -\Gamma^{\mu}_{\lambda\rho}\upsilon^{\lambda}S^{\rho\nu} - \Gamma^{\nu}_{\lambda\rho}\upsilon^{\lambda}S^{\mu\rho} + (p^{\mu}\upsilon^{\nu} - p^{\nu}\upsilon^{\mu}) + \lambda(S^{\mu\kappa}F^{\nu}_{\kappa} - S^{\nu\kappa}F^{\mu}_{\kappa}) \tag{3}$$

where Greek indices take values 0,1,2,3, Latin 1,2,3, τ is an affine parameter across a word line L which is chosen as the proper time of the charged particle, υ^{μ} is the 4-velocity of the charged particle across the world line L, $p^{\mu} = \int T^{0\mu}dV$ are the components of the 4-momentum of the spinning charged particle, $F^{\mu\nu}$ is the electromagnetic tensor, λ is an electromagnetic coupling scalar and $S^{\mu\nu}$ is the spin tensor. Unlike special relativity, p^{μ} and υ^{μ} are not generally proportional to each other. But it is well known that Eqs.(2) and (3) themselves, do not constitute an independent set of equations since they are less than the unknown quantities (3 components of the spin tensor are not determined). Therefore, several supplementary conditions have been used in the literature to remedy this problem [36]. Here we will adopt Dixon's condition [15] e.g. $p_{\mu}S^{\mu\nu} = 0$(center of mass condition).

To find the trajectory of the spinning charged particle, we need to know its 4-velocity. But there are no equations of motion for this purpose. However, we may obtain indirectly a relation between υ^{ν} and p^{ν} from the following equation [1]:

$$\begin{aligned}\upsilon^{\nu} &= N\{u^{\nu} - \frac{1}{2m^2\Delta}R_{\mu\beta\lambda\rho}S^{\lambda\rho}S^{\mu\nu}u^{\beta} + \frac{e}{m^2\Delta}S^{\mu\nu}F_{\mu\beta}u^{\beta} \\ &+ \lambda[\frac{1}{2m^3\Delta}R_{\mu\beta\lambda\rho}S^{\lambda\rho}S^{\mu\nu}S^{\beta\kappa}(F^{\sigma}_{\kappa}u_{\sigma}) - \frac{1}{2m^2\Delta}S^{\mu\nu}S^{\kappa\rho}\nabla_{\mu}F_{\kappa\rho} - \frac{1}{m}S^{\nu\kappa}(F^{\sigma}_{\kappa}u_{\sigma})] \\ &- \frac{\lambda e}{m^3\Delta}S^{\mu\nu}F_{\mu\beta}S^{\beta\kappa}(F^{\sigma}_{\kappa}u_{\sigma})\}\end{aligned} \tag{4}$$

where $p^{\mu} = mu^{\mu}$, $p_{\mu}p^{\mu} = -m^2$, m is the mass of the particle and $G = c = 1$

$$\Delta = 1 + \frac{1}{4m^2}R_{t\mu\lambda\rho}S^{\lambda\rho}S^{t\mu} + \frac{1}{2m^2}eF_{t\mu}S^{t\mu} \tag{5}$$

$$N = \{1 - \Lambda^{(1)}_{\mu}\Lambda^{(1)}_{\alpha}S^{\mu\nu}S^{\alpha}_{\nu} - 2\lambda[\Lambda^{(2)}_{\mu}\Lambda^{(1)}_{\alpha}S^{\mu\nu}S^{\alpha}_{\nu} - (F^{\sigma}_{\kappa}u_{\sigma})\Lambda^{(1)}_{\alpha}S^{\kappa\nu}S^{\alpha}_{\nu}]$$

$$
\begin{aligned}
&- \quad \lambda^2[\Lambda^{(2)}_\mu \Lambda^{(2)}_\alpha S^{\mu\nu} S^\alpha_\nu - 2(F^\sigma_\kappa u_\sigma)\Lambda^{(2)}_\alpha S^{\kappa\nu} S^\alpha_\nu + (F^\sigma_\kappa u_\sigma)(F^\alpha_\beta u_\alpha) S^{\kappa\nu} S^\beta_\nu] \\
&- \quad 2\lambda e \Lambda^{(3)}_\mu \Lambda^{(3)}_\alpha S^{\mu\nu} S^\alpha_\nu - 2e\lambda^2[-\Lambda^{(3)}_\mu \Lambda^{(2)}_\alpha S^{\mu\nu} S^\alpha_\nu + (F^\sigma_\kappa u_\sigma)\Lambda^{(3)}_\mu S^{\mu\nu} S^\kappa_\nu] \\
&+ \quad \lambda^2 e^3 \Lambda^{(3)}_\mu \Lambda^{(3)}_\alpha S^{\mu\nu} S^\alpha_\nu \}^{-1/2}
\end{aligned}
\tag{6}
$$

and

$$\Lambda^{(1)}_x = \frac{1}{2m^2\Delta} R_{x\sigma\lambda\rho} S^{\lambda\rho} u^\sigma + \frac{e}{m^2\Delta} F_{x\sigma} u^\sigma \tag{7}$$

$$\Lambda^{(2)}_x = -\frac{1}{2m^3\Delta} R_{x\sigma\lambda\rho} S^{\lambda\rho} S^{\sigma\kappa} (F^\beta_\kappa u_\beta) + \frac{1}{m^2\Delta} S^{\kappa\rho} \nabla_x F_{\kappa\rho} \tag{8}$$

$$\Lambda^{(3)}_x = \frac{1}{m^3\Delta} F_{x\sigma} S^{\sigma\kappa} (F^\beta_\kappa u_\beta) \tag{9}$$

where x stands for $x = \mu, \alpha$. Upon the consideration of the assumption($\lambda = 0$) (Pomeranskiĭ et al. 2000 and references therein), we neglect particular terms in Eqs.(2),(3) and a simplified covariant model is obtained:

$$\frac{dp^\mu}{d\tau} = -\Gamma^\mu_{\lambda\nu} \upsilon^\lambda p^\nu - \frac{1}{2} R^\mu_{\nu\lambda\rho} S^{\lambda\rho} \upsilon^\nu + e F^\mu_\beta \upsilon^\beta \tag{10}$$

$$\frac{dS^{\mu\nu}}{d\tau} = -\Gamma^\mu_{\lambda\rho} \upsilon^\lambda S^{\rho\nu} - \Gamma^\nu_{\lambda\rho} \upsilon^\lambda S^{\mu\rho} + (p^\mu \upsilon^\nu - p^\nu \upsilon^\mu) \tag{11}$$

$$\upsilon^\nu = N\{u^\nu - \frac{1}{2m^2\Delta} R_{\mu\beta\lambda\rho} S^{\lambda\rho} S^{\mu\nu} u^\beta + \frac{e}{m^2\Delta} S^{\mu\nu} F_{\mu\rho} u^\rho\} \tag{12}$$

with

$$N = [1 - \frac{N_a}{4m^4\Delta^2}]^{-1/2} \tag{13}$$

and

$$
\begin{aligned}
N_a &= (R_{\mu\sigma\lambda\rho} S^{\lambda\rho} S^{\mu\nu} u^\sigma)[R_{\mu\sigma\lambda\rho} S^{\lambda\rho}(\eta_{\nu\beta} + h_{\nu\beta}) S^{\mu\beta} u^\sigma] \\
&- 2e[R_{\mu\sigma\lambda\rho} S^{\lambda\rho}(\eta_{\nu\beta} + h_{\nu\beta}) S^{\mu\beta} u^\sigma](F_{\kappa\gamma} S^{\kappa\nu} u^\gamma) \\
&- 2e(R_{\mu\sigma\lambda\rho} S^{\lambda\rho} S^{\mu\nu} u^\sigma)[F_{\kappa\sigma}(\eta_{\nu\beta} + h_{\nu\beta}) S^{\kappa\beta} u^\sigma] \\
&+ 4e^2[F_{\kappa\sigma}(\eta_{\nu\beta} + h_{\nu\beta}) S^{\kappa\beta} u^\sigma](F_{\kappa\sigma} S^{\kappa\nu} u^\sigma)
\end{aligned}
\tag{14}
$$

In the next section, we discuss the above equations in the linearized theory of gravity.

3 The DS-equations of motion in the linearized theory of gravity

To understand the Eqs.(10-14) in the linearized approximation to general relativity, we decompose the metric in the fashion

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{15}$$

By imposing the condition

$$(h_\mu^\nu - \delta_\mu^\nu h_\rho^\rho)_{;\nu} = 0 \tag{16}$$

we reduce the vacuum field equations to homogeneous wave equations for all components of h_ν^μ. A coordinate transformation can now be effected to reduce the trace h_ν^ν and the mixed components $h_{0\alpha}$, to zero. The gravitational field is then described by a symmetric traceless, divergenceless tensor with two independent space components which, for simplicity, we call $h_1 = h_+$ and $h_2 = h_\times$. Thus, the square of the line element is

$$\begin{aligned} ds^2 &= (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \\ &= -(dx^0)^2 + (1+h_1)dx^2 + (1-h_1)dy^2 + dz^2 + 2h_2 dxdy \end{aligned} \tag{17}$$

where $|h_1, h_2| \ll 1$. We consider a plane GW which is characterized by the wave 3-vector

$$k_g^i = \omega_g(0,0,1) \tag{18}$$

and the two possible states of polarization given by

$$h_1 = h_{10} e^{i(k_g z - \omega_g t)}, \quad h_2 = h_{20} e^{i(k_g z - \omega_g t)} \tag{19}$$

where h_{10}, h_{20} are the amplitudes of the two components of the GW.

We choose the electromagnetic field to be

$$F_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -H_3 & 0 \\ 0 & H_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{20}$$

where the background magnetic field is constant e.g $H^a = (0,0,H^3) = const.$(from now on $H^3 = H$). In this scenario, the metric is still a solution to the Einstein equations in vacuum, because we assume that the energy density of the magnetic field, is approximately zero(i.e. no effect of the magnetic field on either the $\eta_{\mu\nu}$ or $h_{\mu\nu}$).

We intend to discuss the electrodynamics of a spinning point-like charged particle with mass m, with an intrinsic angular momentum, in the presence of a uniform magnetic field across the z-axis, initially at rest with respect to the coordinate system in which the metric (17) is expressed. To achieve this task, a relation between the invariant proper time τ and the coordinate time t is needed. In the absence of external forces, this relation may be found from the expression

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu = -d\tau^2 \tag{21}$$

because in Einstein's theory of gravity, the word lines of classical point particles in curved space times are time-like (see Ref.[33,34]. In the case we have external forces, we have to use the expression

$$d\tau = dt[\frac{1-\upsilon^2}{1-eS^{\mu\nu}F_{\mu\nu}/m^2}]^{1/2} \tag{22}$$

where υ^2 is the total space velocity of the spinning particle. Looking at the Eq.(22) we would like to make the following comments: a)The physical meaning of the Eq.(22) is that relativistic time dilation occurs for a spinning charged particle with non-zero magnetic moments in an external magnetic field. b) The structure of Eq.(22) results from the fact that υ^μ and u^μ differ from each other, where τ is connected with υ^μ and where u^μ is normalized [33,34].

To make some further progress with the Eqs.(10-14), we decompose the particle's components of the 4-velocity, 4-momentum and spin tensor as follows:

$$\upsilon^\mu \simeq \upsilon_0^\mu + \upsilon_1^\mu, \quad p^\mu \simeq p_0^\mu + p_1^\mu, \quad S^{\mu\nu} \simeq S_0^{\mu\nu} + S_1^{\mu\nu} \tag{23}$$

with υ_1^μ, p_1^μ and $S_1^{\mu\nu}$ being of the same order as $h_{\mu\nu}$.

Thus, from Eqs.(10-14) and with the aid of Eqs.(22-23) we obtain the following equations:

3.a) Zero Order Equations

$$\frac{du_0^\mu}{dt} = \frac{e}{m}\eta^{\mu\nu}F_{\nu\beta}\upsilon_0^\beta \tag{24}$$

$$\frac{dS_0^{\mu\nu}}{dt} = m(u_0^\mu\upsilon_0^\nu - u_0^\nu\upsilon_0^\mu) \tag{25}$$

$$\begin{aligned}\upsilon_0^\nu &= [\frac{1}{1-eS^{\mu\nu}F_{\mu\nu}/m^2}]^{1/2}[1-\frac{e^2N_0}{m^4\Delta_0}]^{-1/2}\{u_0^\nu + \\ &+ \frac{e}{m^2\Delta_0}S_0^{\mu\nu}F_{\mu\sigma}u_0^\sigma\}\end{aligned} \tag{26}$$

where

$$\Delta_0 = 1 + \frac{1}{2m^2}eF_{\sigma\mu}S_0^{\sigma\mu} \tag{27}$$

$$N_0 = [F_{\kappa\sigma}S_0^{\kappa\nu}u_0^\sigma][F_{\kappa\sigma}\eta_{\nu\beta}S_0^{\kappa\beta}u_0^\sigma] \tag{28}$$

We assume that the spinning particle initially is at rest, $u_0^\mu = (1,0,0,0), \upsilon_0^\mu = (1,0,0,0)$; then the condition $p_{\mu(0)}S_0^{\mu\nu} = 0$, implies that the zero order electric moments of the spin-tensor vanish e.g $S_0^{0\nu} = 0$. After some straightforward calculations

and with the aid of Eqs.(22),(27) and (28) we find; $\Delta_0 = 1 - \frac{eH}{m}(\frac{S_0^{12}}{m})$ and, for the system (25), the following zero-order equations are derived:

$$S_0^{12} = constant, S_0^{13} = constant, S_0^{23} = constant \tag{29}$$

3.b) First Order Equations

$$\frac{dp_1^\mu}{dt} = -\Gamma^\mu_{\lambda\nu}\upsilon_0^\lambda p_0^\nu - \frac{1}{2}\eta^{\mu\kappa}R_{\kappa\nu\lambda\rho}S_0^{\lambda\rho}\upsilon_0^\nu + e[\eta^{\mu\kappa}F_{\kappa\beta}\upsilon_1^\beta + h^{\mu\kappa}F_{\kappa\beta}\upsilon_0^\beta] \tag{30}$$

$$\frac{dS_1^{\mu\nu}}{dt} = -\Gamma^\mu_{\lambda\rho}\upsilon_0^\lambda S_0^{\rho\nu} - \Gamma^\nu_{\lambda\rho}\upsilon_0^\lambda S_0^{\mu\rho} + [p_0^\mu\upsilon_1^\nu + p_1^\mu\upsilon_0^\nu - p_0^\nu\upsilon_1^\mu - p_1^\nu\upsilon_0^\mu] \tag{31}$$

$$\begin{aligned} \upsilon_1^\nu &= [\frac{1}{1 - eS^{\mu\nu}F_{\mu\nu}/m^2}]^{1/2}\{\frac{1}{\Delta_0^2}[2e^2N_{1a} - eN_{1b} - 4e^2\frac{N_0\Delta_1}{\Delta_0}][u_0^\nu + \\ &+ \frac{e}{m^2\Delta_0}F_{\kappa\sigma}S_0^{\kappa\nu}u_0^\sigma] \\ &+ [1 - \frac{e^2N_0}{m^4\Delta_0^2}]^{-1/2}[u_1^\nu - \frac{e}{2m^2\Delta_0}R_{\mu\sigma\lambda\rho}S_0^{\lambda\rho}S_0^{\mu\nu}u^\sigma \\ &+ \frac{e}{m^2\Delta_0}F_{\kappa\sigma}(S_0^{\kappa\nu}u_1^\sigma + S_1^{\kappa\nu}u_0^\sigma) - \frac{e\Delta_1}{m^2\Delta_0^2}F_{\kappa\sigma}S_0^{\kappa\nu}u_0^\sigma]\} \end{aligned} \tag{32}$$

where

$$\Delta_1 = \frac{1}{4m^2}R_{\sigma\mu\lambda\rho}S_0^{\lambda\rho}S_0^{\mu\sigma} + \frac{e}{2m^2}F_{\sigma\mu}S_1^{\sigma\mu} \tag{33}$$

$$\begin{aligned} N_{1a} &= (F_{\kappa\sigma}S_0^{\kappa\nu}u_0^\sigma)[F_{\kappa\sigma}\eta_{\nu\beta}(S_0^{\kappa\beta}u_1^\sigma + S_1^{\kappa\beta}u_0^\sigma)] \\ &+ (F_{\kappa\sigma}\eta_{\nu\beta}S_0^{\kappa\beta}u_0^\sigma)[F_{\kappa\sigma}(S_0^{\kappa\nu}u_1^\sigma + S_1^{\kappa\nu}u_0^\sigma)] \end{aligned} \tag{34}$$

and

$$N_{1b} = -2eR_{\mu\sigma\lambda\rho}[S_0^{\lambda\rho}\eta_{\nu\beta}S_0^{\mu\beta}u_0^\sigma(F_{\kappa\beta}S_0^{\kappa\nu}u_0^\beta) - S_0^{\lambda\rho}S_0^{\mu\nu}u_0^\sigma(F_{\kappa\sigma}\eta_{\nu\beta}S_0^{\kappa\beta}u_0^\sigma) \tag{35}$$

Because of the assumptions made in paragraph (3.a) and the form of the metric (17), the Eqs.(28),(33),(34) and (35) give the following results; $N_0 = N_{1a} = N_{1b} = 0, \Delta_1 = \frac{1}{2}\{h_{1,zz}[(\frac{S_0^{13}}{m})^2 - (\frac{S_0^{23}}{m})^2] + 2h_{1,zz}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\} - \frac{eH}{m}(\frac{S_0^{12}}{m})$

$$\frac{du_1^0}{dt} = 0, \quad \frac{du_1^3}{dt} = 0 \tag{36}$$

$$\frac{du_1^1}{dt} + \Omega\upsilon_1^2 = \frac{1}{2m}[h_{1,tz}S_0^{13} + h_{2,tz}S_0^{23}] \tag{37}$$

$$\frac{du_1^2}{dt} - \Omega\upsilon_1^1 = -\frac{1}{2m}[h_{1,tz}S_0^{23} - h_{2,tz}S_0^{13}] \tag{38}$$

$$\frac{dS_1^{0\nu}}{dt} = m(\upsilon_1^\nu - u_1^\nu), \quad \frac{dS_1^{12}}{dt} = 0 \tag{39}$$

$$\frac{dS_1^{13}}{dt} = -\frac{1}{2}[h_{1,t}S_0^{13} + h_{2,t}S_0^{23}] \tag{40}$$

and

$$\frac{dS_1^{23}}{dt} = \frac{1}{2}[h_{1,t}S_0^{23} - h_{2,t}S_0^{13}] \tag{41}$$

The Eqs.(23) give:

$$\upsilon_1^0 = \frac{u_1^0}{\Delta_0^{1/2}} \tag{42}$$

$$\upsilon_1^1 = \frac{u_1^1}{\Delta_0^{3/2}}(1 - 2\Omega\frac{S_0^{12}}{m}) + \frac{1}{2\Delta_0^{3/2}}[h_{1,tz}\frac{S_0^{12}}{m}\frac{S_0^{23}}{m} - h_{2,tz}\frac{S_0^{12}}{m}\frac{S_0^{13}}{m}] \tag{43}$$

$$\upsilon_1^2 = \frac{u_1^2}{\Delta_0^{3/2}}(1 - 2\Omega\frac{S_0^{12}}{m}) + \frac{1}{2\Delta_0^{3/2}}h_{1,tz}\frac{S_0^{12}}{m}[\frac{S_0^{13}}{m} - \frac{S_0^{23}}{m}] \tag{44}$$

$$\begin{aligned} \upsilon_1^3 &= \frac{u_1^3}{\Delta_0^{1/2}} + \frac{1}{\Delta_0^{3/2}}\{h_{1,tz}[(\frac{S_0^{13}}{m})^2 - (\frac{S_0^{23}}{m})^2] + 2h_{2,tz}(\frac{S_0^{23}}{m})^2\} \\ &- \frac{\Omega}{\Delta_0^{3/2}}[u_1^2\frac{S_0^{13}}{m} - u_1^1\frac{S_0^{23}}{m}] \end{aligned} \tag{45}$$

where comma means partial differentiation and $\Omega = \frac{eH}{m}$ is the cyclotron frequency. We solve the Eqs.(36-45) and find:

$$u_1^0 = const., \quad u_1^3 = const. \tag{46}$$

$$\begin{aligned} u_1^1 &= \frac{\Delta_0^{3/2}}{D_+}\{[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)][\cos(At) - \cos(\omega_g t)] \\ &+ [B_{10}\cos(k_g z) - B_{20}\sin(k_g z)][\sin(At) - \sin(\omega_g t)]\} \end{aligned} \tag{47}$$

and

$$\begin{aligned} u_1^2 &= \frac{\Delta_0^{3/2}}{D_+}\{[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)]\sin(At) \\ &- [B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\cos(At)\} \\ &+ \frac{\sin(\omega_g t)}{D_-}[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\ &+ \frac{\cos(\omega_g t)}{D_+}[B_{10}\cos(k_g z) - B_{20}\sin(k_g z)] \end{aligned} \tag{48}$$

where

$$B_{10} = \frac{\omega_g k_g}{2}[h_{10}(\frac{S_0^{13}}{m}) + h_{20}(\frac{S_0^{23}}{m})] - \frac{\Omega\omega_g k_g}{2\Delta_0^{3/2}} h_{10}(\frac{S_0^{12}}{m})[\frac{S_0^{13}}{m} - \frac{S_0^{23}}{m}] \quad (49)$$

$$B_{20} = -\frac{\omega_g k_g}{2}[h_{10}(\frac{S_0^{23}}{m}) - h_{20}(\frac{S_0^{13}}{m})][1 - \frac{\Omega}{\Delta_0^{3/2}}(\frac{S_0^{12}}{m})] \quad (50)$$

$$A = \frac{\Omega}{\Delta_0^{3/2}}[1 - 2\Omega(\frac{S_0^{12}}{m})] \quad (51)$$

and

$$D_{\pm} = \Omega - 2\Omega^2(\frac{S_0^{12}}{m}) \pm \omega_g \Delta_0^{3/2} \quad (52)$$

From Eqs.(42-45) and Eqs.(46-48) we find: $v_0^1 = u_0^1 = const.$ and

$$\begin{aligned}
v_1^1 &= [\frac{1 - 2\Omega(\frac{S_0^{12})}{m}}{D_+}]\{[\cos(At) - \cos(\omega_g t)][B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&+ [\sin(At) + \sin(\omega_g t)][B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\} \\
&+ \frac{\omega_g k_g}{2\Delta_0^{3/2}}[h_{10}(\frac{S_0^{12}}{m})(\frac{S_0^{23}}{m}) - h_{20}(\frac{S_0^{12}}{m})(\frac{S_0^{13}}{m})]e^{i(k_g z - \omega_g t)} \qquad (53)
\end{aligned}$$

$$\begin{aligned}
v_1^2 &= [\frac{1 - 2\Omega(\frac{S_0^{12})}{m}}{D_+}]\{\sin(At)[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&- [\cos(At) - \cos(\omega_g t)][B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\} \\
&+ [\frac{1 - 2\Omega(\frac{S_0^{12}}{m})}{D_-}]\sin(\omega_g t)[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&+ \frac{\omega_g k_g}{2\Delta_0^{3/2}}(\frac{S_0^{12}}{m})[(\frac{S_0^{13}}{m}) - (\frac{S_0^{23}}{m})]h_{10}e^{i(k_g z - \omega_g t)} \qquad (54)
\end{aligned}$$

$$\begin{aligned}
v_1^3 &= \frac{u_1^3}{\Delta_0^{1/2}} - \frac{\Omega}{D_+}(\frac{S_0^{13}}{m})\{\sin(At)[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&- \cos(At)[B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\} \\
&+ \frac{\Omega}{D_+}(\frac{S_0^{23}}{m})\{[\cos(At) - \cos(\omega_g t)][B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&+ [\sin(At) + \sin(\omega_g t)][B_{10}\cos(k_g z) - B_{20}\sin(k_g z)]\} \\
&- \frac{\Omega}{D_-}(\frac{S_0^{13}}{m})\sin(\omega_g t)[B_{10}\sin(k_g z) + B_{20}\cos(k_g z)] \\
&- \frac{\Omega}{D_+}(\frac{S_0^{13}}{m})\cos(\omega_g t)[B_{10}\cos(k_g z) - B_{20}\sin(k_g z)] \\
&+ \frac{\omega k_g}{2\Delta_0^{3/2}}\{h_{10}[(\frac{S_0^{13}}{m})^2 - (\frac{S_0^{23}}{m})^2] + 2h_{20}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\}e^{i(k_g z - \omega_g t)} \qquad (55)
\end{aligned}$$

Derivation of the spinning's particle trajectories: For such a particle, with υ^μ given from the above equations, the trajectories with respect to the coordinate system we consider and the initial conditions $t=0, X_1^\mu(t=0)=0$ are:

$$X_1^0 = \frac{u_1^0 t}{\Delta_0^{3/2}} \tag{56}$$

$$\begin{aligned} X_1^1 &= \frac{1-2\Omega(\frac{S_0^{12})}{m}}{D_+}\{[\frac{\sin{(At)}}{A} - \frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}] \\ &+ [\frac{1-\cos{(\omega_g t)}}{\omega_g} + \frac{1-\cos{(At)}}{A}][B_{10}\cos{(k_g z)} - B_{20}\sin{(k_g z)}]\} \\ &- \frac{ik_g}{2\Delta_0^{3/2}}e^{ik_g z}[1-e^{-i\omega_g t}][h_{10}(\frac{S_0^{12}}{m})(\frac{S_0^{23}}{m}) - h_{20}(\frac{S_0^{12}}{m})(\frac{S_0^{13}}{m})] \end{aligned} \tag{57}$$

$$\begin{aligned} X_1^2 &= \frac{1-2\Omega(\frac{S_0^{12}}{m})}{D_+}\{[\frac{\sin{(\omega_g t)}}{\omega_g} - \frac{\sin{(At)}}{A}][B_{10}\cos{(k_g z)} - B_{20}\sin{(k_g z)}] \\ &+ [\frac{1-\cos{(At)}}{A}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}]\} \\ &+ [\frac{1-2\Omega(\frac{S_0^{12}}{m})}{D_+}][\frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}] \\ &- \frac{ik_g}{2\Delta_0^{3/2}}e^{ik_g z}[1-e^{-i\omega_g t}]h_{10}(\frac{S_0^{12}}{m})[(\frac{S_0^{13}}{m}) - (\frac{S_0^{23}}{m})] \end{aligned} \tag{58}$$

and

$$\begin{aligned} X_1^3 &= \frac{u_1^3 t}{\Delta_0^{3/2}} + [\frac{\Omega}{D_+}(\frac{S_0^{13}}{m})]\{[\frac{\sin{(At)}}{A} - \frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\cos{(k_g z)} - B_{20}\sin{(k_g z)}] \\ &- [\frac{1-\cos{(At)}}{A}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}]\} \\ &- [\frac{\Omega}{D_-}(\frac{S_0^{13}}{m})][\frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}] \\ &+ [\frac{\Omega}{D_+}(\frac{S_0^{23}}{m})]\{[\frac{\sin{(At)}}{A} - \frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)} + B_{20}\cos{(k_g z)}] \\ &+ [\frac{1-\cos{(At)}}{A} + \frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\cos{(k_g z)} - B_{20}\sin{(k_g z)}]\} \\ &+ \frac{ik_g}{2\Delta_0^{3/2}}e^{ik_g z}[1-e^{-i\omega_g t}]\{h_{10}[(\frac{S_0^{13}}{m})^2 - (\frac{S_0^{23}}{m})^2] + 2h_{20}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\} \end{aligned} \tag{59}$$

Derivation of the spin equations: Integrating Eqs.(39-41) we find the components of the $S_1^{\mu\nu}$ tensor which are:

$$
\begin{aligned}
\frac{S_1^{01}}{m} &= \frac{1}{D_+}[1-2\Omega(\frac{S_0^{12}}{m})-\Delta_0^{3/2}]\{[\frac{\sin{(At)}}{A}-\frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&+ [\frac{1-\cos{(At)}}{A}+\frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}]\} \\
&- \frac{ik_g}{2\Delta_0^{3/2}}(\frac{S_0^{12}}{m})e^{ik_g z}[1-e^{-i\omega_g t}][h_{10}(\frac{S_0^{23}}{m})-h_{20}(\frac{S_0^{13}}{m})] \qquad (60)
\end{aligned}
$$

$$
\begin{aligned}
\frac{S_1^{02}}{m} &= \frac{1}{D_+}[1-2\Omega(\frac{S_0^{12}}{m})-\Delta_0^{3/2}]\{[\frac{1-\cos{(At)}}{A}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&- [\frac{\sin{(At)}}{A}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}]\} \\
&+ [1-2\Omega(\frac{S_0^{12}}{m})-\Delta_0^{3/2}]\{[\frac{1-\cos{(\omega_g t)}}{\omega_g D_-}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&+ [\frac{\sin{(\omega_g t)}}{\omega_g D_+}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}]\} \\
&- \frac{ik_g}{2\Delta_0^{3/2}}e^{ik_g z}[1-e^{-i\omega_g t}]h_{10}(\frac{S_0^{12}}{m})[(\frac{S_0^{13}}{m})-(\frac{S_0^{23}}{m})] \qquad (61)
\end{aligned}
$$

$$
\begin{aligned}
\frac{S_1^{03}}{m} &= \frac{\Omega}{D_+}(\frac{S_0^{23}}{m})\{[\frac{\sin{(At)}}{A}-\frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&+ [\frac{1-\cos{(At)}}{A}+\frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}]\} \\
&- \frac{\Omega}{D_+}(\frac{S_0^{13}}{m})\{[\frac{1-\cos{(At)}}{A}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&- [\frac{\sin{(At)}}{A}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}]\} \\
&- \frac{\Omega}{D_-}(\frac{S_0^{13}}{m})[\frac{1-\cos{(\omega_g t)}}{\omega_g}][B_{10}\sin{(k_g z)}+B_{20}\cos{(k_g z)}] \\
&- \frac{\Omega}{D_+}(\frac{S_0^{13}}{m})[\frac{\sin{(\omega_g t)}}{\omega_g}][B_{10}\cos{(k_g z)}-B_{20}\sin{(k_g z)}] \\
&- \frac{ik_g}{2\Delta_0^{3/2}}e^{ik_g z}[1-e^{-i\omega_g t}]\{h_{10}[(\frac{S_0^{13}}{m})^2-(\frac{S_0^{23}}{m})^2]+2h_{20}(\frac{S_0^{13}}{m})(\frac{S_0^{23}}{m})\} \qquad (62)
\end{aligned}
$$

$$\frac{S_1^{12}}{m} = constant \qquad (63)$$

$$\frac{S_1^{13}}{m} = \frac{1}{2}[h_{10}(\frac{S_0^{13}}{m})+h_{20}(\frac{S_0^{23}}{m})]e^{ik_g z}[1-e^{-i\omega_g t}] \qquad (64)$$

and

$$\frac{S_1^{23}}{m} = -\frac{1}{2}[h_{10}(\frac{S_0^{23}}{m}) - h_{20}(\frac{S_0^{13}}{m})]e^{ik_g z}[1 - e^{-i\omega_g t}] \tag{65}$$

From the Eqs.(60-65) we verify that because of the GW we have non-zero first order electric and magnetic moments of the spinning charged particle and in the absence of the GW, all these components disappear. The electric moments particularly exhibit resonances because in the denominators of the Eqs.(60-62) appear the expressions $\Delta_0 = 1 - \frac{eH}{m}\frac{S_0^{12}}{m}$, $A = \frac{\Omega}{\Delta_0^{3/2}}[1 - 2\Omega(\frac{S_0^{12}}{m})]$ and $D_\pm = \Omega - 2\Omega^2(\frac{S_0^{12}}{m}) \pm \omega_g \Delta_0^{3/2}$, which become zero for certain values of the Larmor frequency $\Omega = \frac{eH}{m}$, the ratio $\frac{S_0^{12}}{m}$ and angular frequency of the GW, ω_g. We notify that while the electric moments of the spinning charged particle exhibit such an interesting behavior, the magnetic moments are independent from the magnetic field and the S_0^{12} zero order component of the spin tensor. Also, for the same reasons mentioned above the Eqs.(53-55) exhibit resonaces. In the neighborhood of those resonaces the charged spinning particle gains energy from the GW and accelerates radiating. The above comments become more plausible by examining a special case of the Eqs.(60-65) and Eqs.(53-55) in the appendix .

4. Discussion

Dealing with the interaction of a GW with a spinning particle in the presence of a uniform magnetic field in the linearized theory of general relativity, we found the following results:

1) In the case where the GW and magnetic field are across the z axis, the components of the 4-velocity, 4-momentum and the spin tensor $S^{\mu\nu}$, exhibit resonance at $\Omega = (\frac{S_0^{12}}{m})^{-1}$ ($\Delta_0 = 0$). Due to the co-existence of the constant magnetic field with GW, a strong coupling between the frequency $\Omega = \frac{eH}{m}$ and the magnetic moment S_0^{12} of the charged spinning particle occur. This coupling gives rise to the above resonance.

Also, for the same reasons mentioned above, some other resonances appear in the Eqs.(47-48),Eqs.(49-51), Eqs.(53-55),Eqs.(56-59) and Eqs.(60-65) which are solutions to 4-order polynomial in terms of Ω; $D_\pm = 0 \Rightarrow 4\Omega^4(\frac{S_0^{12}}{m})^2 - 4\Omega^3[\omega_g^2(\frac{S_0^{12}}{m})^3 - 4\frac{S_0^{12}}{m})] + \Omega^2[1 - 3\omega_g^2(\frac{S_0^{12}}{m})^2] + 3\omega_g^2(\frac{S_0^{12}}{m})\Omega - \omega_g^2 = 0$.

2) It is interesting to notify that in the absence of the GW, the magnetic field and the components of the spin tensor decouple and the magnetic resonances disappear. In this case, where the GW does not exist(see ref.33 and references there in), the motion of a spinning charged point-particle of mass m and charge q is described in an 4-dimensional Minkowski space time by its position $X^\mu(t)$, defining the particle's world line, its 4-velocity u^μ, which is tangent to the world-line and its polarization tensor $D_{\mu\nu}(t)$, an antisymmetric 4-tensor which combines the intrinsic magnetic dipole moment $\mathbf{M}$(a pseudo 3-vector) with the intrinsic electric dipole moment $\mathbf{d}$ (a real 3-vector) at every given point of the world-line through the relations $D_{ij} = \frac{1}{c}\epsilon_{ijk}M_k$ and $-iD_{i4} = d_i$ where $(i, j, k) = 1, 2, 3$. In the absence of external fields, the intrinsic dipole moments are found from the values of $\mathbf{M}$ and $\mathbf{d}$ (rest frame of the free particle). Usually we are interested in charged particle with no intrinsic electric dipole moment in the rest frame of the free particle. This may expressed by the condition $D_{\mu\nu}u^\nu = 0$. On the other hand, the polarization tensor is relate to an intrinsic angular momentum tensor $S_{\mu\nu}$(spin tensor) through the expression $D_{\mu\nu} = (q/mc)S_{\mu\nu}$. From the above mentioned equations we have the condition $S_{\mu\nu}u^\nu = 0$. When this relation holds, $S_{\mu\nu}$ is space-like with only 3 non-zero components in the rest frame of the free particle e.g. $S_{ij}^{(0)} = \epsilon_{ijk}s^k$ and $S_{i0}^{(0)} = 0$. Besides, we have to point out that in the case of the unperturbed Minkowski space-time the classical spin is introduced somehow indirectly, via the electromagnetic polarization tensor, because the empirical meaning of classical magnetic and electric dipole moments is clear.

3) In the case that the GW does exist and in the limit of the high frequency approximation[37,38], the charged particle which initial is at rest, starts to have a combination of an orbital and spinning motion, described by the Eqs.(56-59) and Eqs.(60-65), respectively. The spin tensor $S^{\mu\nu} = S_0^{\mu\nu} + S_1^{\mu\nu}$, exhibits electric and magnetic moments eventhough initially had magnetic moments only. Because of the Eqs.(39), the electric moments exhibit the same resonances as the components of the

4-momentum and in the neighborhood of those resonaces energy is transferred from the GW to the spinning particle. The magnetic moments do not depend neither on the magnetic field nor on the component S^{12}. Under these circumstances one could hope to detect such GW.

A possible astrophysical environment where the interaction studied in this paper maybe be of relevance is the binary neutron star merger. In this scenario, two magnetized neutron stars merge, forming (if the equation of state allows it) a very massive, differentially rotating object and a possible low-mass disk around it (the object could survive for hundreds of seconds before collapsing to a black hole [39]. The magnetosphere of this object will be rotating rapidly and be filled with plasma, while near the object, gravitational waves of large amplitude will be emitted. It would be interesting to study the conditions under which the interaction studied in the present paper could lead to observable phenomena during such a binary neutron star merger.

To make some further comments related to possible astrophysical application of the Eqs.(51-65), we have to consider the Pauli-Lubanski covariant spin vector formula $S_\sigma = \frac{1}{2}\epsilon_{\rho\mu\nu\sigma}u^\rho S^{\mu\nu}$, which gives $S_0^{12} = S_0^3$, $S_0^{13} = -S_0^2$, $S_0^{23} = S_0^1$ and assume, for simplicity, that e.g. $S_0^3 = S_0^1 = S$, while $S_0^2 = 0$, then some typical values for this scenario of some astrophysical importance may be met when, for example, the amplitude of the GW is $h \approx 10^{-10}$, $H \approx 10^6 G$, and for an electron one has roughly $S \approx 10^{-13}$m [1]. Such conditions can be found around various compact objects, for example near neutron stars which posses a magnetic field of the order $10^8 - 10^{12}G$ and emit GW due to glitches or rotational instabilities excited by accretion (see Ref.[40,41,42])

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Appendix

In the appendix we will present a special case of the Eqs.(53-55) and Eqs.(60-67).

We assume $z = 0$ and for further simplicity we obtain $h_{10} = h_{20} = h_0$ and $S_0^{13} = 0$. Because of the assumption $c = G = 1$, $k_g = \omega_g$. Now Eqs.(53-52) read:

$$
\begin{aligned}
v_1^1 &= \frac{\omega_g^2}{2D_+} h_0 (\frac{S_0^{23}}{m})[1 - 2\Omega(\frac{S_0^{12}}{m})]\{[\cos(\omega_g t) - \cos(At)][1 - \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ [\sin(\omega_g t) + \sin(At)][1 + \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&+ h_0 \sin(\omega_g t) \frac{\omega_g^2}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})(\frac{S_0^{12}}{m}) \qquad (66)
\end{aligned}
$$

$$
\begin{aligned}
v_1^2 &= -\frac{\omega_g^2}{2D_+} h_0 (\frac{S_0^{23}}{m})[1 - 2\Omega(\frac{S_0^{12}}{m})]\{\sin(At)][1 - \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ [\cos(At) - \cos(\omega_g t)][1 + \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- h_0 \sin(\omega_g t) \frac{\omega_g^2}{2D_-}(\frac{S_0^{23}}{m})[1 - 2\Omega(\frac{S_0^{12}}{m})][1 - \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&- h_0 \cos(\omega_g t) \frac{\omega_g^2}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})(\frac{S_0^{12}}{m}) \qquad (67)
\end{aligned}
$$

$$
\begin{aligned}
v_1^3 &= \frac{u_1^3}{\Delta_0^{1/2}} + \frac{\Omega\omega_g}{2D_+} h_0 (\frac{S_0^{23}}{m})^2 \{[\cos(\omega_g t) - \cos(At)][1 - \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ [\sin(\omega_g t) + \sin(At)][1 + \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- h_0 \cos(\omega_g t) \frac{\omega_g^2}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})^2 \qquad (68)
\end{aligned}
$$

Also the Eqs.(60-67) become:

$$
\begin{aligned}
\frac{S_1^{01}}{m} &= \frac{\omega_g^2}{2D_+} h_0 (\frac{S_0^{23}}{m})[1 - 2\Omega(\frac{S_0^{12}}{m}) - \Delta_0^{3/2}]\{[\frac{\sin(\omega_g t)}{\omega_g} - \frac{\sin(At)}{A}][1 - \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ [\frac{1 - \cos(\omega_g t)}{\omega_g} + \frac{1 - \cos(At)}{A}][1 + \frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- h_0 \sin(\omega_g t) \frac{\omega_g}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})(\frac{S_0^{12}}{m}) \qquad (69)
\end{aligned}
$$

$$
\begin{aligned}
\frac{S_1^{02}}{m} &= -\frac{\omega_g^2}{2D_+}h_0(\frac{S_0^{23}}{m})[1-2\Omega(\frac{S_0^{12}}{m})-\Delta_0^{3/2}]\{\frac{1-\cos{(At)}}{A}[1-\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ \frac{\sin{(At)}}{A}[1+\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- \frac{\omega_g^2}{2}h_0(\frac{S_0^{23}}{m})[1-2\Omega(\frac{S_0^{12}}{m})-\Delta_0^{3/2}]\{\frac{1-\cos{(\omega_g t)}}{\omega_g D_-}[1-\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&- \frac{\sin{(\omega_g t)}}{\omega_g D+}[1+\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- h_0\sin{(\omega_g t)}\frac{\omega_g}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})(\frac{S_0^{12}}{m}) \qquad (70)
\end{aligned}
$$

$$
\begin{aligned}
\frac{S_1^{03}}{m} &= \frac{\Omega\omega_g^2}{2D_+}h_0(\frac{S_0^{23}}{m})^2\{[\frac{\sin{(\omega_g t)}}{\omega_g}-\frac{\sin{(At)}}{A}][1-\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})] \\
&+ [\frac{1-\cos{(\omega_g t)}}{\omega_g}+\frac{1-\cos{(At)}}{A}][1+\frac{\Omega}{\Delta_0^{2/3}}(\frac{S_0^{12}}{m})]\} \\
&- h_0\sin{(\omega_g t)}\frac{\omega_g}{2\Delta_0^{2/3}}(\frac{S_0^{23}}{m})^2 \qquad (71)
\end{aligned}
$$

$$
\frac{S_1^{12}}{m} = const. \qquad (72)
$$

$$
\frac{S_1^{13}}{m} = -\frac{S_1^{23}}{m} = \frac{1}{2}h_0(\frac{S_0^{23}}{m})[1-\cos{(\omega_g t)}] \qquad (73)
$$

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